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Event-Symmetric Physics

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Abstract

I examine various aspects of event-symmetric physics such as phase changes, symmetry breaking and duality by studying a number of simple toy-models.

Keywords

quantum gravity, discrete space-time, event-symmetric space-time, pre-geometry model, symmetric group, spontaneously broken symmetry, simplicial lattice field theory, dynamical triangulation, random graphs, matrix model, Lie algebra, supersymmetry, duality

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To Rachel

(because she is the first to believe it!)

Event-Symmetric Space-Time

More than anybody else John Wheeler has promoted the idea that physics must be derived from some deeper pregeometric theory in which space-time structure arises as an aspect of a more fundamental one [?]. It is very difficult to know how to approach the task of building such a theory but there have at least been some worthy attempts which can serve as a source of ideas [?].

Many of them take some guiding principle as a basis for constructing a theory. It may be causality, topology or spin structure for example. But in precious few cases does symmetry enter as a basic necessity. This is surprising since symmetry has been the most useful tool in constructing successful theories of physics this century. The difficulty may simply be that nobody has been able to see how symmetry can be used in a pregeometric theory.

My own belief is that the symmetry so far discovered in nature is just the tiny tip of a very large iceberg most of which is hidden beneath a sea of symmetry breaking. With the pregeometric theory of *event-symmetric physics* I hope to unify the symmetry of space-time and internal gauge symmetry into one huge symmetry. I hope that it may be possible to go even further than this. Through dualities of the type being studied in string theory it may be possible to include the permutation symmetry under exchange of identical particles into the same unified structure. Ultimately we may come to understand the origins of so much symmetry in terms of some metaphysical theory of theories (see e.g. [49]).

The real domain of the event-symmetric formalism seems to be in string theory. I have explored the construction of event-symmetric string field theories elsewhere [50] and hope to return to it later. In this article I explore a number of much simpler event-symmetric toy models which provide some useful insight into the nature of event-symmetric physics.

The theory of Event-Symmetric space-time is a discrete approach to quantum gravity [51]. The exact nature of space-time in this scheme will only become apparent in the solution. Even the number of space-time dimensions is not set by the formulation and must be a dynamic result. It is possible that space-time will preserve a discrete nature at very small length scales. Quantum mechanics must be reduced to a minimal form. The objective is to find a statistical or quantum definition of a partition function which reproduces a unified formulation of known and hypothesised symmetries in physics and then worry about states, observables and causality later.

Suppose we seek to formulate a lattice theory of gravity in which diffeomorphism invariance takes a simple and explicit discrete form. At first glance it would seem that only translational invariance can be adequately represented in a discrete form on a regular lattice. This overlooks the most natural generalisation of diffeomorphism invariance in a discrete system.

Diffeomorphism invariance requires that the action should be symmetric under any differentiable 1-1 mapping on a D dimensional manifold M_D . This is represented by the diffeomorphism group $diff(M_D)$. On a discrete space we could demand that the action is symmetric under any permutation of the discrete space-time events ignoring continuity altogether. Generally we will use the term *Event-Symmetric* whenever an action has an invariance under the Symmetric Group $S(\mathcal{U})$ over a large or infinite set of “events” \mathcal{U} . The symmetric group is the group of all possible 1-1 mappings on the set of events with function composition as the group multiplication. The cardinality of events on a manifold of any number of dimensions is \aleph_1 . The number of dimensions and the topology of the manifold is lost in an event-symmetric model since the symmetric groups for two sets of equal cardinality are isomorphic.

Event-symmetry is larger than the diffeomorphism invariance of continuum space-time.

$$diff(M_D) \subset S(M_D) \simeq S(\aleph_1) \quad (1)$$

If a continuum is to be restored then it seems that there must be a mechanism of spontaneous symmetry breaking in which event-symmetry is replaced by a residual diffeomorphism invariance. The mechanism will determine the number of dimensions of space. It is possible that a model could have several phases with different numbers of dimensions and may also have an unbroken event-symmetric phase. Strictly speaking we need to define what is meant by this type of symmetry breaking. This is difficult since there is no order parameter which can make a qualitative distinction between a broken and unbroken phase.

The symmetry breaking picture is not completely satisfactory because it suggests that one topology is singled out and all others discarded by the symmetry breaking mechanism but it would be preferred to retain the possibility of topology change in quantum gravity. It might be more accurate to say that the event-symmetry is not broken. This may not seem to correspond to observation but notice that diffeomorphism invariance of space-time is equally inevident at laboratory scales. Only the Poincare invariance of space-time is easily seen. This is because transformations of the metric must be included

to make physics symmetric under general coordinate changes. It is possible that some similar mechanism hides the event-symmetry. I will continue to use the language of symmetry breaking even if it may not be strictly correct.

It is possible to make an argument based on topology change that space-time *must* be taken as event-symmetric in Quantum Gravity. Wheeler was the first to suggest that topology changes might be a feature of quantum geometrodynamics [1]. Over the past few years the arguments in favour of topology change in quantum gravity have strengthened see e.g. [52]. If we then ask what is the correct symmetry group in a theory of quantum gravity under which the action is invariant, we must answer that it contains the diffeomorphism group $diff(M)$ for any manifold M which has a permitted topology. Diffeomorphism groups are very different for different topologies and the only reasonable way to include $diff(M)$ for all M within one group is to extend the group to include the symmetric group $S(\aleph_1)$. There appears to be little other option unless the role of space-time symmetry is to be abandoned altogether.

There is another theory which would benefit from a formulation in which fields have non-local interactions independent of distance of the type postulated in event-symmetric theories. The theory shows that fine-tuning of the constants of nature could be explained in such circumstances [53].

It is unlikely that there would be any way to distinguish a space-time with an uncountable number of events from space-time with a dense covering of a countable number of events so it is acceptable to consider models in which the events can be labelled with positive integers. The symmetry group $S(\aleph_1)$ is replaced with $S(\aleph_0)$. In practice it may be necessary to regularise to a finite number of events N with an $S(N)$ symmetry and take the large N limit while scaling parameters of the model as functions of N .

Having abandoned diffeomorphisms we should ask whether there can remain any useful meaning of topology on a manifold. A positive answer is provided by considering discrete differential calculus on sets and finite groups [44].

In some of the more physically interesting models the symmetry appears as a sub-group of a larger symmetry such as the orthogonal group $O(N)$. It is also sufficient that the Alternating group $A(N)$ be a symmetry of the system since it contains a smaller symmetric group.

$$S(N) \subset A(2N) \tag{2}$$

The definition of the term event-symmetric could be relaxed to include systems with invariance under the action of a group which has a homomor-

phism onto $S(N)$. This would include, for example, the braid group $B(N)$ and, of course, quantum groups such as $SL_q(N)$.

Renormalisation and the continuum limit must also be considered but it is not clear what is necessary or desired as renormalisation behaviour. In asymptotically free quantum field theories with a lattice formulation such as QCD it is normally assumed that a continuum limit exists where the lattice spacing tends to zero as the renormalisation group is applied. In string theories, however, the theory is perturbatively finite and the continuum limit of a discrete model cannot be reached with the aid of renormalisation. It is possible that it is not necessary to have an infinite density of events in space-time to have a continuum or there may be some alternative way to reach it, via a q -deformed non-commutative geometry for example.

It stretches the imagination to believe that a simple event-symmetric model could be responsible for the creation of continuum space-time and the complexity of quantum gravity through symmetry breaking, however, nature has provided some examples of similar mechanisms which may help us accept the plausibility of such a claim and provide a physical picture of what is going on.

Consider the way in which soap bubbles arise from a statistical physics model of molecular forces. The forces are functions of the relative positions and orientations of the soap and water molecules. The energy is a function symmetric in the exchange of any two molecules of the same kind. The system is consistent with the definition of event-symmetry since it is invariant under exchange of any two water or soap molecules and therefore has an $S(N) \otimes S(M)$ symmetry where N and M are the number of water and soap molecules. Under the right conditions the symmetry breaks spontaneously to leave a diffeomorphism invariance on a two dimensional manifold in which area of the bubble surface is minimised.

Events in the soap bubble model correspond to molecules rather than space-time points. Nevertheless, it is a perfect mathematical analogy of event-symmetric systems where the symmetry breaks in the Riemannian sector to leave diffeomorphism invariance in two dimensions as a residual symmetry. Indeed the model illustrates an analogy between events in event-symmetric space-time and identical particles in many-particle systems. The models considered further are more sophisticated than the molecular models. However, the analogy between particles and space-time events remains a useful one.

It might be asked what status this approach affords to events themselves. Events are presented as fundamental entities almost like particles.

Event orientated models are sometimes known as Whiteheadian [54] but Wheeler preferred to refer to a space-time viewed as a set of events without a geometric structure as a “bucket of dust” [2, 3]. In some of the models we will examine it appears as if events are quite real, perhaps even detectable. In other models they are more metaphysical and it is the symmetric group that is more fundamental. Indeed the group may only arise as a subgroup of a matrix group and the status of an event is then comparable to that of the component of a vector. Then again in the discrete string models we will see that events have the same status as strings.

The concept of event-symmetric space-time fits well into the framework of non-commutative geometry. It has been shown [55] that by defining differential geometry on a space-time consisting of a manifold times a discrete set of points it is possible to give a geometric interpretation of theories such as the Electro-Weak Standard Model [56] in which the Higgs field arises naturally from the generalised connection. If we could start from a non-commutative geometry defined on just a large discrete set of points with an event-symmetric formulation then this would make sense of these model building techniques.

A number of Event-Symmetric models will be described in this paper. Some of these can best be understood as statistical theories with a partition function defined for a real positive definite action.

$$Z = \int e^{-S} \quad (3)$$

Others can only be considered as quantum theories for which the action need not be positive definite provided the partition function is well defined

$$Z = \int e^{iS} \quad (4)$$

It is not always clear when such an integral should be considered well defined. For example the action,

$$S = x^2 - y^2 \quad (5)$$

gives a well defined quantum partition function in the two variables (x, y) but if the variables are transformed by a 45 degree rotation to (u, v) , the action becomes

$$S = 2uv \quad (6)$$

for which the integral is not well defined.

It might be safer to consider only positive definite actions and assume that in a physically valid theory, the only difference between the statistical event-symmetric model and the quantum one should be a factor of i against the action in the exponential. We might expect that in the statistical version the Event-Symmetry will break to give Riemannian space-time with a Euclidean signature metric while in the quantum version it breaks to give the physical Einsteinian space-time theory with Lorentzian signature metric.

But is that realistic, after all, continuum Lagrangian densities for field theories in Minkowski space-time are made non-positive definite by the signature of the metric? It is not clear what conditions should be placed on the form of an event-symmetric action to ensure a well defined tachyon free quantum theory which produces dynamically the correct Lorentz signature. Even in continuum theories this is an interesting question and it is believed that a Lorentzian signature is preferred for certain theories in 4 and 6 dimensions. [57, 58, 59].

Event-Symmetric Ising Models

The simplest event-symmetric model is the event-symmetric Ising model. This consists of a large number N of ferromagnets represented by spin variables

$$s_a = +1 \text{ or } -1 \text{ for } (a = 1, \dots, N) \quad (7)$$

Each spin interacts equally with every other spin according to the action,

$$S = \beta \sum_{a < b} s_a s_b \quad (8)$$

This has $S(N)$ invariance since it is symmetric in spin permutations and an additional Z_2 invariance under global spin reversal. Solving this model is not difficult. The partition function is

$$Z = \sum_{\{s_a\}} e^{-S} \quad (9)$$

Write this as a sum over states with K negative spins and $N - K$ positive spins.

$$Z = \sum C_K^N \exp((\beta/N)[(N/2)(N-1) - 2K(N-K)]) \quad (10)$$

In the large N limit this can be written as an integral over a variable

$$p = K/N \quad (11)$$

$$Z = \int_0^1 dp \exp\{N(\bar{\beta}[1/2 - 2p(1-p)] - p \ln(p) - (1-p)\ln(1-p))\} \quad (12)$$

In this equation we have scaled β as a function of N such that $\bar{\beta} = \beta N$ is kept constant.

The function in the exponential has one minimum at $p = 1/2$ for $\beta \leq 1$ and two minima for $\beta > 1$. The large N limit forces the system into these minima so there is a second order phase transition at $\beta = 1$ with the Z_2 spin symmetry broken above. The $S(N)$ event-symmetry is not broken in this model.

Although such a model seems quite trivial there is some interest in generalisations where the Z_2 symmetry is replaced with unitary matrix groups [60].

For the gauged version the spins are placed on event links. There are therefore $(1/2)N(N-1)$ spins

$$s_{ab} = +1 \text{ or } -1, a < b \quad (13)$$

And the action is now a sum over triangles formed from three links

$$S = \beta \sum_{a < b < c} s_{ab}s_{bc}s_{ac} \quad (14)$$

This model again has an $S(N)$ event-symmetry but the Z_2 symmetry is now a gauge symmetry. This is already too complicated to solve exactly by any obvious means.

The most interesting thing that can be said about this model is that it is dual to a model of surfaces which can be compared to string world sheets. Let T be the set of all possible triangles with vertices in the set of events. i.e.,

$$T = (a, b, c) : a < b < c \quad (15)$$

then,

$$Z = \sum_{\{s_{ab}\}} \prod_{(a,b,c) \in T} (\cosh \beta + s_{ab}s_{bc}s_{ac} \sinh \beta) \quad (16)$$

$$= \cosh \beta^{1/6N(N-1)(N-2)} \sum_{R \in 2^T} \tanh \beta^{|R|} \sum_{\{s_{ab}\}} \prod_{(a,b,c) \in R} s_{ab}s_{bc}s_{ac} \quad (17)$$

The inner sum over the product is zero except when the subset R of triangles contains each link variable an even number of times. Such a subset can be considered a surface formed from the triangles. It may be made up of several pieces and it may cross itself at links. The outer sum can then be replaced

with a sum over surfaces B and if the number of triangles in B is interpreted as its area $A(B)$ then an effective action is left given by

$$S' = \ln(\tanh\beta)A(B) \quad (18)$$

This is analogous to the Area action for a first quantised bosonic string but is defined on an event-symmetric lattice instead of a D dimensional continuous target space.

Molecular Models

Insight into event-symmetric statistical physics models and the possibilities for symmetry breaking can be gained from molecular models. In general a molecular model describes a large number N of molecules given by their position vector X_a and orientation matrices O_a in a D dimensional Euclidean hyperspace. For simplicity kinetic energy is discarded and the interactions are described by an energy potential,

$$E = V(\{X_a\}, \{O_a\}) \quad (19)$$

The potential should tend rapidly to a constant at large distances in order to suppress long range interactions, and should be invariant under global translations and rotations. Furthermore the potential should be invariant under exchange of any two molecules so that the description event-symmetric can be justified. An analogy then exists between the molecular model and a model of an event-symmetric space-time in which events correspond to molecules.

The simplest possibility is to model space-time as a critical solid [61]. For a suitable action symmetric in exchange of molecules they can model a critical solid at a second order melting phase transition. This gives rise dynamically to what might be interpreted as a D dimensional curved manifold. In this case the number of dimensions is predetermined and it is difficult to see how the space-time could form different topologies. The event-symmetry is broken in the solid phase since the molecules settle into a lattice configuration leaving a residual translation symmetry. Near the phase transition a scaling behaviour might be observed with a larger residual symmetry in the critical limit. In a gas phase the model would be fully event-symmetric.

More persuasive models might be constructed by attempting to simulate a molecular model of soap film bubbles. A single species of molecule in

D -dimensional hyperspace with an orientation dependent energy potential favouring alignment should be sufficient. In such a model the molecules would tend to form lattices on lower d -dimensional hyper-surfaces at low temperatures.

To make this more concrete we shall look for a suitable energy function. Take it to be a sum of potential energies between molecule pairs in $D = 3$ space.

$$S = \beta E = \sum_{ab} V(X_a, X_b, O_a, O_b) \quad (20)$$

Spatial symmetry is ensured if the potential is a function of the following scalar invariants,

$$r_{ab} = |X_a - X_b| \quad (21)$$

$$\cos(\theta_{ab}) = \hat{i} \cdot O_a(X_a - X_b)/r_{ab}, 0 \leq \theta_{ab} \leq \pi \quad (22)$$

Where \hat{i} is a unit vector in the axis of the molecule. A likely looking possibility in terms of these invariants is,

$$S = \beta \sum_{ab} [2r_{ab}^{-1} - \sin^2(\theta_{ab})] e^{-r_{ab}} \quad (23)$$

The minimum energy configuration for two molecules is when they are a distance $r = 1 + \sqrt{3}$ apart and are oriented perpendicular $\theta_{ab} = \pi/2$ to the line which joins them. With many particles, the minimum energy state is achieved when they are packed into a triangular lattice in a $d = 2$ plane. The spacing can be computed numerically to be $r = 2.13$. The configuration is stable against movement of a molecule out of the plane.

At low temperatures (large β) the molecules would stay near the lattice positions so there would be a solid crystalline phase. At higher temperature the molecules might be expected to flow in the plane simulating a liquid phase. There should be a phase transition between the solid and liquid phase at which scaling behaviour might be observed. At higher temperatures the bubble would evaporate into a gas phase and the full event-symmetry would be restored.

The most interesting part of the phase diagram might be the transition from the crystalline to fluid phase. Similar transitions have been studied numerically in the context of lattice gravity where there might be an interesting transition between fixed triangulations and random triangulations on surfaces [62].

The benefit of bubble models is that surfaces with different topologies are possible and diffeomorphism invariance is a possibility as a residual symmetry. It is probable that the $D = 3, d = 2$ model can be generalised to higher dimensions.

The analogy between statistical molecular models forming soap films and event-symmetric models forming space-time can be a very useful one to help us visualise the physics of event-symmetric theories. We can really imagine space-time evaporating at high temperature for example. There are, however, some important differences which must be born in mind: A molecular model is formed as an embedding in a higher dimensional space whereas the most interesting event-symmetric models are defined in some kind of Machian void; In molecular models the discrete objects are always hard objects which can be detected individually whereas in event-symmetric models events may be only a bases of an algebra with no existence as individual objects. Finally the molecular models are models in statistical physics with no time evolution whereas a physical event-symmetric model must be a full quantum theory even if time is not an exact concept.

Molecular models are well understood in terms of equilibrium thermodynamics under changes of temperature and pressure. It may be possible to define a phase diagram of event-symmetric theories in a similar way. It would then be possible to think of the formation of space-time as a condensation process. If space-time behaves like molecular models then it may be possible to go from the broken phase to the unbroken phase of gravity just as it is possible to go from a gas to a liquid at high pressure without passing through a phase transition. There might also be physical significance of critical points. Of course it might not be possible to define temperature and pressure in an event-symmetric model.

Symmetric Random Graph Models

Since we are looking for some kind of spontaneous symmetry breaking in which the number of dimensions is dynamically determined it makes sense to investigate systems on which we can attempt to define dimensionality. The simplest such structure would be a random graph in which N nodes are randomly pairwise connected by up to $1/2N(N - 1)$ links [12, 63, ?, 39, 46]. An event-symmetric action is a function of the connections which is invariant under any permutation of nodes. For example, actions defined as functions of the total number of links and the total number of triangles in a graph

would be event-symmetric.

The principle is that on a graph we can define dimensionality from its connectivity. For a given node we can define a function $L(s)$, the number of nodes which can be reached by taking at most s steps along links. If $L(s)$ has a power law on an infinite graph,

$$L(s) \rightarrow s^D \text{ as } s \rightarrow \infty \quad (24)$$

then the graph has dimension D . It may also be possible to determine dimensionality from topology of a finite graph [64] or, if the links are bidirectional, the topology can be derived from an analysis of posets [42].

If a suitable mechanism of symmetry breaking is effected on the system the graphs generated statistically from the action may have some finite dimension. The number of dimensions could differ from one phase of the system to another. There could also be phases in which the event-symmetry is unbroken and the number of dimensions can be considered infinite.

The random graph models are similar in some ways to the random lattice models of quantum gravity but are much simpler since there is no need to apply constraints which select the topology in the formulation. Instead the sum is over all discrete topologies. It is also much easier to ensure that an action is positive definite.

Wheeler was one of the first to think about this sort of space-time model [3]. He Likened a random graph to a sewing machine stitching together a space-time. Wheeler found that such models did not appeal to his taste in simplicity. Nevertheless they are a useful starting point for exploration of event-symmetric space-times even if they are unphysical.

Since there are no other symmetries to guide our choice of action we might consider heuristic criteria to contrive an action which might exhibit spontaneous symmetry breaking of the event-symmetry. As a first guess it might be reasonable to consider an action which favours triangles but disfavors links. The action can be written in terms of link variables l_{ab}

$$l_{ab} = 1 \text{ if the nodes } a \text{ and } b \text{ are linked, } = 0 \text{ otherwise} \quad (25)$$

Define

$$V_a = \sum_b l_{ab} \quad (26)$$

$$T_a = \sum_{b,c} l_{ab} l_{bc} l_{ac} \quad (27)$$

I.e. V_a is the valence of node a and T_a is the number of triangles in the graph which have a vertex at a .

$$S = - \sum_a [(\beta/N^2)T_a - (\alpha/N^2)V_a^2] \quad (28)$$

A simple mean field analysis can be performed where each link is connected with a probability p . Then

$$T_a = N^2 p^3 \quad (29)$$

$$V_a = Np \quad (30)$$

Taking into account that the number density of states as a function of p this gives an effective action of,

$$S = N[p \ln(p) + (1-p) \ln(1-p) - \beta p^3 + \alpha p^2] \quad (31)$$

This suggests a phase transition along approximately $\beta/\alpha = 1$ with p close to one for $\beta > \alpha$ and p close to zero for $\beta < \alpha$

Further mean field analysis of this model and other similar models is possible. An extension to the treatment given here would be to consider a mean field analysis of the situation where the graph breaks down into small isolated parts. Linkage between nodes within each part can be given a probability p while linkage between nodes in different parts can be given a probability q . A mean field analysis for a particular Event-Symmetric action might suggest that an asymmetric phase existed with q small and p close to one. It is possible that this could be taken as a signal that other forms of Symmetry Breaking were a possibility for that action

Numerical simulations could also be used to look for evidence of Event-Symmetry breaking. It may be possible to construct models in this way which have residual structures with finite dimensional symmetries.

In fact there is one very simple random graph model in which the symmetry breaks to one dimension. This is given by the action,

$$S = \beta \sum_a (V_a - 2)^2 \quad (32)$$

At high β the model forces exactly two links to meet at each vertex of the graph. I.e. it must break down into rings which can be considered as one dimensional spaces.

Dynamical Triangulations

Lattice studies of pure gravity start from the Regge Calculus [65] in which space-time is “triangulated” into a simplicial complex. The dynamical variables are the edge lengths of the simplices. In 4 dimensions an action which reduces to the usual Einstein Hilbert action in the continuum limit can be defined as a sum over hinges in terms of facet areas A_h and deficit angles δ_h which can be expressed in terms of the edge lengths.

$$S = \sum_h k A_h \delta_h \quad (33)$$

The model can be studied as a quantised system and this approach has had some limited success in numerical studies [66, 67, 68, 69].

A variation of the Regge calculus is to use dynamical triangulations. An action with fixed edge lengths but random triangulations [70] is given by,

$$S = -\kappa_4 N_4 + \kappa_0 N_0 \quad (34)$$

The partition function is formed from a sum over all possible triangulations of the four-sphere. N_4 is the number of four simplices in the triangulation and N_0 is the number of vertices. The constant κ_4 is essentially the cosmological constant while κ_0 is the gravitational coupling constant. Random triangulations of space-time appear to work somewhat better than the Regge Calculus with a fixed triangulation.

It is possible to construct event-symmetric models which reduce to dynamical triangulations of manifolds as limiting cases. This is certainly an interesting prospect given the provisional success of dynamical triangulations as models of Riemannian sector quantum gravity in numerical simulations.

Two dimensional manifolds can be broken down into triangles so we define a triangle variable t_{abc} which takes the value 1 or 0 according to whether or not the three events a , b and c are the vertices of a triangle in the triangulation. In an event-symmetric model these are simply dynamic variables defined for any three events analogous to the link variables of the random graph models. The following constraints are applied,

$$t_{abc} = t_{bac} = t_{acb} \quad (35)$$

$$t_{aac} = 0 \quad (36)$$

The number of triangles meeting at an edge defined by two events a, b is

$$L_{ab} = \sum_c t_{abc} \quad (37)$$

In a dynamical triangulation of a manifold this must be everywhere either zero or two. We can define an action,

$$S = \beta \sum_{ab} L_{ab}^2 (L_{ab} - 2)^2 \quad (38)$$

In the high β limit this forces the triangles to form a triangulation of some manifold but there is nothing to ensure that the manifold is connected or that it has to be oriented. The sum must be over all topologies.

It is possible to force the manifolds to be oriented. This can be done by allowing the value for t_{abc} to be 0, 1 or -1 , with the constraints

$$t_{abc} = -t_{bac} = -t_{acb} \quad (39)$$

An edge in the triangulation must have a contribution from a negative triangle and a positive triangle to ensure the surface is oriented. A suitable action can also be contrived for this case. It is important to make a restriction to oriented manifolds since otherwise parity violation would not be possible.

Such models can easily be generalised to give simplicial decompositions of higher D dimensional manifolds by defining variables with D indices which are fully antisymmetric. This justifies the assertion that dynamical triangulations are limiting cases of event-symmetric systems provided a sum over all topologies is included.

These are very crude models but there are a couple of important lessons to be learnt here. The first is that event-symmetric models which incorporate higher dimensional objects than the simple events and links which appeared in the random graph models seem to have better potential for forming space-time structure. There are interesting models with field variables defined on simplex like structures of arbitrarily high dimension which might be very promising in this respect.

The second lesson is that continuum space-time models of quantum gravity which include a suitable weighted sum over topologies can be seen as limiting cases of event-symmetric models. Heuristically we might conclude that the sum over topologies factors out the diffeomorphic structure of theories with diffeomorphism invariance on manifolds leaving a completely event-symmetric theory. This may be physically important if 4 dimensional quantum gravity at low energy can thus be seen as a limit of a more complete event-symmetric theory in which space-time dimension is not precisely defined.

Event-Symmetric field Theory

The random graphs are interesting as models of space-time but ultimately we are interested in modelling field theories. It is conceivable that field theories with continuous variables could somehow arise out of theories with discrete variables but if we are to see gauge symmetries of the type found in Yang-Mills Theories represented in an exact discrete form at a more fundamental level then continuous variables must be used.

The simplest event-symmetric field theory would be given by a scalar field ϕ_a defined on events a . We might define an action of the form,

$$S = \sum_a m\phi_a^2 + \sum_{a<b} (\phi_a - \phi_b)^2 + \sum_a g\phi_a^n \quad (40)$$

If $n > 2$ is even and g is positive then a statistical field theory can be defined with partition function,

$$Z = \int \exp(-S) d^N \phi \quad (41)$$

For odd n the action is not positive definite but a quantum field theory is well defined with the partition function,

$$Z = \int \exp(iS) d^N \phi \quad (42)$$

It is useful to look at the perturbation theory of such models. First the non-interacting $g = 0$ case should be solved. The quadratic part of the action takes the form,

$$S = \sum_{a,b} E(m + (N - 1), -1)_{ab} \phi_a \phi_b \quad (43)$$

where the notation $E(d, e)$ is used to denote an *event-symmetric matrix* which has the value d for each diagonal element and the value e for each off diagonal element. The propagator will be given by the inverse of this matrix which is easily found to be,

$$E(m + (N - 1), -1)^{-1} = E(m + 1, 1) / [m(m + N)] \quad (44)$$

This would be singular in the case where $m = 0$.

The propagators in the interacting case can be represented as a sum over Feynman diagrams which take the form of fixed valence graphs. I.e. the graphs have exactly n edges joined at each vertex. For each vertex there

is a sum over events and a factor g . For each edge there is a propagator factor $1/[m(m+N)]$. If we take m to be small and ignore accidental symmetry factors then the contribution of each graph is,

$$(gN)^{N_0}(mN)^{-N_1} \quad (45)$$

Where N_0 is the number of nodes and N_1 is the number of edges.

$$2N_1 = nN_0 \quad (46)$$

A sum over fixed valence graphs can be considered as another type of event-symmetric random graph model. This simple type can be solved. There are more interesting versions constructed from Ising models on the nodes of fixed valence graphs which can also be studied analytically by relating them to the perturbation theory of event-symmetric scalar field theories [71, 72, 73]. From our point of view gauged Ising models on fixed valence random graphs might be even more interesting.

If the ultimate aim is to produce event-symmetric models of real physics then it will be necessary to introduce further symmetries such as gauge symmetry. The Event-symmetric Ising gauge model can be combined with a random graph model giving a model with link variables which can take three values -1, 0 or +1. Such models are interesting to study for event-symmetry breaking because the duality transformation can still be applied to give a dual model of strings on a random graph.

To go further the Z_2 gauge symmetry can be extended to gauge symmetry of other groups such as $U(1)$, $SU(3)$ etc. The link variables then takes values zero or an element of the group. Such models represent a kind of gauge glass [74, 49]. We shall see that there are more unified ways to combine event-symmetry and gauge symmetries.

For the symmetry to break in the way we desire, i.e. leaving a finite dimensional topology, the events will have to organise themselves into some arrangement where there is an approximate concept of distance between them perhaps defined by correlations between field variables. Matrix elements linking events which are separated by large distances would have to be correspondingly small. Only variables which are localised with respect to the distance could have significant values.

Field theory can be extended further than placing field variables on just sites and links between sites. They can also be attached to higher dimensional cells or simplices such as triangles and tetrahedrons. This can be understood as the field theory extension of dynamical triangulations. It may

be easier to analyse dimensional phase transitions in such a context. Useful work in this context had been produced by Jourjine and Vanderseypen who express field theory on cell complexes in the mathematical language of homology and cochains [18, 19, 21, 35].

Random Matrix Models

An important class of event-symmetric model places field variables A_{ab} on links joining all pairs of events (a, b) . A suitable action must be a real scalar function of these variables which is invariant under exchange of any two events.

The link variables A_{ab} can be regarded as the elements of a matrix A . If the direction of the link is irrelevant the matrix can be taken to be either symmetric or anti-symmetric. If there are no self links the diagonal terms are zero so it is natural to make the matrix anti-symmetric.

$$A_{ab} = -A_{ba} \quad (47)$$

A possible four link loop action is

$$S = \beta \sum_{a,b,c,d} A_{ab}A_{bc}A_{cd}A_{da} + m \sum_{a,b} A_{ab}A_{ab} \quad (48)$$

$$= \beta \text{Tr}(A^4) + m \text{Tr}(A^2) \quad (49)$$

which is an invariant under $O(N)$ similarity transformations on the matrix.

The symmetric group $S(N)$ is incorporated as a sub-group of $O(N)$ represented by matrices with a single one in each row or column and all other elements zero in such a way that the matrix permutes the elements of any vector it multiplies. This suggests that in general we should consider actions which are functions of the traces of powers of the matrix A . The same idea can be extended to unitary groups by using complex variables for hermitian matrices or symplectic groups by using quaternions.

This is an appealing scheme since it naturally unifies the $S(N)$ symmetry, which we regard as an extension of diffeomorphism invariance, with gauge symmetries. If the symmetry broke in some miraculous fashion then it is conceivable that the residual symmetry could describe quantised gauge fields on a quantised geometry.

Consider for example a discrete gauge $SO(10)$ symmetry on a hypercubic lattice of $N = M^4$ points. The full symmetry group $\text{Lat}(SO(10), M)$

is generated by the gauge group $SO(10)^N$ and the lattice translation and rotation operators. A matrix representation of this group in $10N \times 10N$ orthogonal matrices can be constructed from the action of the group on a 10 component field placed on lattice sites. The group is therefore (isomorphic to) a sub-group of an orthogonal group.

$$Lat(SO(10), M) \subset O(10N) \quad (50)$$

We can imagine a mechanism by which the $O(N)$ symmetry of a matrix model broke to leave a residual $Lat(SO(10), M)$ symmetry. It seems highly unlikely, however, that such an exact form of symmetry breaking could arise spontaneously.

This type of random matrix model has been extensively studied in the context where N is interpreted as the number of colours or flavours. (see [75, 76]) The event-symmetric paradigm suggests an alternative interpretation in which N is the number of events times the number of colours [16].

This unification of space-time and internal gauge symmetries might be compared with the similar achievement of Kaluza-Klein theories in which the symmetry is also extended and assumed broken. Here the symmetry is much larger and could be compared with a Kaluza-Klein theory which had an extra dimension for each field variable [14].

One interesting result for matrix models which is responsible for them attracting so much interest is that the perturbation theory of an $SU(N)$ matrix model in the large N double scaling limit is equivalent to two dimensional gravity or a $c = 0$ string theory [77, 78].

To see this observe that the Feynman diagrams form graphs with nodes of valence v corresponding to terms in the action given by the trace of matrices to the power v . The edges meeting at a given node are cyclically ordered in correspondence to the multiplication order of the matrices in the trace. Given this ordering it is possible to form a surface from the graph. Faces formed from edges are identified by following loops of edges round the graph in such a way that the next edge in the loop is consistent with the cyclic ordering of the edges at each node.

The diagrams are thus in one to one correspondence with facetting of surfaces with restricted vertex valency. The sum over diagrams for surfaces of any given topology defines a field theory on the phase space of facet decompositions of that surface. It is found that the contributions from a given diagram is in fact a topological invariant of the surface. This universality is explained by a correspondence between primitive moves changing

the decomposition and the fact that the matrix algebra is associative and semi-simple [79].

Many generalisation to multi-matrix models have been studied. In describing the general forms for actions that we can allow for these models we must apply a certain locality principle as well as the gauge invariance. The action must be restricted to forms in which it is the sum of terms which are functions of the trace of matrix expressions and which do not separate into products of two or more such scalar quantities. For example if there are two matrices A and B defining the field variables then the action could contain terms such as,

$$tr(ABAB) \tag{51}$$

but not,

$$tr(AB)^2 \tag{52}$$

or

$$tr(A)tr(B) \tag{53}$$

This locality condition is important when selecting suitable actions for models which might exhibit dimensional symmetry breaking, since otherwise the broken phase would have long range interactions.

Random Tensor Models

The matrix models have several possible generalisations to tensor models and models with fermions. In each case the action can be a function of any set of scalars derived from the tensors by contraction over indices, with the indices ranging over space-time events.

In tensor models it is often useful to associate tensors which have certain symmetry constraints with geometric objects having the same symmetry in such a way that the indices correspond to vertices of the object. For example a rank 3 tensor which is symmetric under cyclic permutations of indices

$$T_{abc} = T_{bca} \tag{54}$$

can be associated with a triangle joining the three vertices a , b and c . Often models of interest use fully anti-symmetric rank- d tensors which can be associated with an oriented d -simplex.

If symmetry breaking is going to separate events then locality is important. Happily there is a sense in which we can define local interactions

independently of any symmetry breaking mechanisms within the general context of tensor models.

In each of the models we have looked at there are field variables which have an association with one or more events. In matrix models the matrix elements A_{ab} are associated with two events indexed by a and b . They represent an amplitude for the connection of those two events as linked neighbours in space-time. In tensor models a tensor of rank r is likewise associated with r events. When symmetry breaking occurs we expect the events to somehow spread themselves over a manifold. A field variable associated with events which are not neighbours on the manifold should be physically insignificant, this will usually mean that it is very small. Field variables which are associated with a local cluster of events can be large and are significant in the continuum limit. Two such field variables which are localised around different parts of the manifold should not be strongly correlated. They must therefore not appear in the same interaction term of the action unless multiplied by some other small field variable.

This heuristic picture leads to a definition of locality in which interaction terms are excluded if they separate into the product of two parts which do not share events. More precisely we can define an *interaction graph* corresponding to any interaction term which has a node for each variable in the term. Two nodes are linked if the variables are associated with at least one event in common. We then say that the model satisfies the *weak locality principle* if all interaction graphs are connected. We will also say that it satisfies the *strong locality principle* if every pair of nodes is connected in all graphs. I.e. they are triangles, tetrahedrons or higher dimensional simplexes.

As an example, a matrix model with terms given by the traces of powers of the matrix,

$$I_n = \text{tr}(A^n) \tag{55}$$

are weakly local because the graphs are n -sided polygons with possibly other links. If the model includes only powers up to the third then it is strongly local.

It is reasonable to expect that physical event-symmetric field theories would have to be at least weakly local. There seems to be no special reason to demand that a theory should be strongly local but it is notable that this condition often reduces the number of possible interaction terms from infinity down to one or two without seeming to exclude the most interesting models.

There have also been interesting studies based on rank three tensors where the perturbation theory describes the joining of tetrahedral simplices to build a three dimensional space [80, 81, 82]. However, these models do not exhibit the same universality properties that make the matrix models so powerful. This fault has been corrected by Boulatov who replaces tensors with functions on groups and defines an action which generates 3 dimensional lattice models [83, 84, 85].

Supersymmetric Models

It would be an obvious next step to generalise to supersymmetric matrix models [86, 87, 88, 89]. So far we have matrix models based a families of groups such as $O(N)$, $SU(N)$ or $Sp(N)$. Tensor representations and invariants can be used to construct models with commuting variables, anticommuting variables or both. Similarly we can define models based on supermatrix groups of which there are also several families such as $SU(L|K)$ and $OSp(L|K)$. For analysis and classification of supergroups see [90].

One simple super event-symmetric model has an anti-hermitian matrix A of commuting variables

$$\overline{A}_{ab} = -A_{ba} \quad (56)$$

and a vector ψ of anti-commuting variables. A suitable action could be,

$$S = m(2i\overline{\psi}_a\psi_a + A_{ab}A_{ba}) \quad (57)$$

$$+ \beta(3\overline{\psi}_aA_{ab}\psi_b - iA_{ab}A_{bc}A_{ca}) \quad (58)$$

As well as $U(N)$ invariance this is invariant under a super-symmetry transform with an infinitesimal anticommuting parameter ϵ_b ,

$$\delta A_{ab} = \overline{\epsilon}_b\psi_a - \epsilon_a\overline{\psi}_b \quad (59)$$

$$\delta\psi_a = i\epsilon_bA_{ab} \quad (60)$$

$$\delta\overline{\psi}_a = i\overline{\epsilon}_bA_{ba} \quad (61)$$

It is necessary to confess that this model is flawed because the super-symmetry is not closed. It can be completed with the inclusion of a single scalar variable but this spoils its locality.

A more general class of models can be constructed from superhermitian matrices which take a block diagonal form,

$$S = \begin{pmatrix} A & B \\ iB^\dagger & C \end{pmatrix} \quad (62)$$

where A is a hermitian K by K matrix of commuting variables, B is a K by L matrix of anticommuting variables and C is a hermitian L by L matrix of commuting variables. The supertrace is defined as

$$sTr(S) = Tr(A) - Tr(C) \quad (63)$$

Actions defined with terms expressed as the supertrace of powers of the matrices are invariant under a $U(K|L)$ super-symmetry. This can be interpreted as an event-symmetric model with two types of event since the supergroup has a sub-group isomorphic to $S(K) \otimes S(L)$. If there is also a vector with components on events in the model then it would have commuting variables on one type of event and anticommuting on another. It is possible to interpret this as an indication that events themselves have either bosonic or fermionic statistics in this model.

It is encouraging that supersymmetric generalisations of matrix models can be so easily constructed on event-symmetric space-time. Demanding supersymmetry helps reduce our choice of actions but not actually very much. There are still many different possibilities like the above which can be constructed from contractions over tensor representations of supersymmetry groups. With such models we would hope to find examples of symmetry breaking where the residual symmetry included space-time supersymmetry but these models are special cases of matrix or tensor models so they will not be more successful as a scheme for dimensional symmetry breaking.

Spinor Models

If it is not possible to break event-symmetry with simple tensor models then it is necessary to investigate models with spinor representations or models with tensors of unlimited rank.

The advantage of spinors is that the dimension of the representations increases exponentially with N . For a model using a finite number of tensor representations the dimension is only polynomial in N .

A simple model would have an $O(N)$ symmetry and a Dirac spinor Ψ representation with $2^{N/2}$ anticommuting components. An invariant action can be constructed using the gamma matrices in the spirit of a Gross-Neveu model [91].

$$S = im\bar{\Psi}\Psi + \beta\bar{\Psi}\Gamma_a\Psi\bar{\Psi}\Gamma_a\Psi \quad (64)$$

This model can be solved by introducing a bosonic variable σ_a to remove the 4th degree term

$$S = im\bar{\Psi}\Psi + 2\beta\bar{\Psi}\Gamma_a\Psi\sigma_a - \beta\sigma_a\sigma_a + (N/2)\ln(2\pi\beta) \quad (65)$$

The fermionic variables can then be integrated giving the determinant of a matrix whose eigenvalues are easily derived.

$$Z = (2\pi\beta)^{N/2} \int d\sigma^N |imI + 2\beta\Gamma_a\sigma_a| \exp(-\beta\sigma^2) \quad (66)$$

$$Z = (2\pi\beta)^{N/2} \int d\sigma^N (4\beta^2\sigma^2 + m^2)^M \exp(-\beta\sigma^2) \quad (67)$$

$$M = 2^{N/2-1} \quad (68)$$

which can be reduced to an integral over one variable,

$$Z = \beta^{N/2} \Gamma(N/2)^{-1} \int_0^\infty d\sigma (4\beta^2\sigma^2 + m^2)^M \sigma^{N-1} \exp(-\beta\sigma^2) \quad (69)$$

By integrating completely we destroy any possibility of symmetry breaking. It is necessary to introduce some kind of symmetry breaking term and rework. There are various terms which could be considered but the simplest is a vector term

$$S_1 = \bar{\Psi}\Gamma_a\Psi v_a \quad (70)$$

By $O(N)$ invariance a vector term can be rotated to have just one component. So add a term to the action of the form

$$S_1 = \bar{\Psi}\Gamma_1\Psi v \quad (71)$$

then,

$$Z = \beta^{N/2} \Gamma((N-1)/2)^{-1} \int_0^\infty d\sigma \int_{-\infty}^\infty d\sigma_1 (4\beta^2\sigma^2 + (2\beta\sigma_1 + v)^2 + m^2)^M \sigma^{N-2} \exp[-\beta(\sigma^2 + \sigma_1^2)] \quad (72)$$

The integrand has two maxima in σ_1 which dominate the integral. The asymmetry introduced by the vector term causes a shift from one maxima to the other and dynamically breaks the symmetry. Taking the limit $a \rightarrow 0$ indicates that spontaneous breaking of symmetry can arise.

The result is symmetry breaking from $O(N)$ to $O(N-1)$. Although this is far from being what we are looking for, a mechanism which selects one event would be interesting if that event could be identified as the initial event of the universe! c.f. [92, 93].

A better understanding of this kind of model can be found from a different version in terms of staggered fermions [94] on a 2^N lattice, i.e. an N -dimensional hypercube. A real or complex fermionic variable ψ_i is placed on each lattice site i and interactions are described in terms of matrices of sign factors Γ_a^{ij} linking edges of the N dimensional hypercube. Where (i, j) is not an edge of the hypercube in direction a the matrix component is zero. Elsewhere the matrices are taken equal to ± 1 on each edge such that the product round any square plaquette is -1 and such that the matrices are antisymmetric.

There are many ways to fulfill this but they are all equivalent under some transformation of sign changes on the fermionic variables. The matrices satisfy the usual anticommutation relations for Dirac's gamma matrices in Euclidean N -dimensional space.

The action is

$$S = im\overline{\psi}_i\psi_i + \beta\psi_i\Gamma_a^{ij}\psi_j\overline{\psi}_i\Gamma_a^{ij}\overline{\psi}_j \quad (73)$$

Despite being formulated on a lattice this has an exact $SO(N)$ invariance which can be seen explicitly by reducing the representation to a family of antisymmetric tensors

$$(\alpha, \alpha_a, \alpha_{ab}, \alpha_{abc}, \dots) \quad (74)$$

The scalar α is placed on one corner of the lattice. The vector components are placed on the N sites which are linked to that corner according to the direction of the link. In general the components of the rank- r tensor are placed on the C_r^N sites which can be reached through r links from the corner. With a suitable choice of the sign factors in the gamma matrices we get

$$\psi_i\Gamma_a^{ij}\psi_j = \alpha\alpha_a + \alpha_b\alpha_{ba} + \alpha_{bc}\alpha_{bca} + \dots \quad (75)$$

This makes the $SO(N)$ invariance explicit but since the corner was an arbitrary choice the symmetry should be larger. In fact the model has a $Spin(N) \otimes Spin(N)$ invariance.

The tensor formulation is interesting because it allows us to interpret the model in terms of interactions between fields defined on sets of events. E.g. a component α_{abc} can be regarded as a field variable assigned to the set of events $\{a, b, c\}$. There are 2^N field variables corresponding to the number of possible subsets of the N events.

This is only one step away from a field theory defined on string like objects which pass through a sequence of events. Only the ordering is missing.

Finally we note that the most basic representations of the Braid group $B(N)$ are also defined to act on a space of dimension 2^N . This suggests

that quantum group versions of event-symmetric field theories with $S(N)$ replaced by $B(N)$ might be possible.

The above observations were the original inspiration behind generalised event-symmetric models involving string field theories and quantum groups.

Simplex Models

In [50] I constructed groups over a basis of discrete strings in event-symmetric space-time. Another class of groups closely related to the string groups is based on sets of discrete events where the order does not matter except for a sign factor which changes according to the signature of permutations,

$$((a|b|c)) = -((b|a|c)) \quad (76)$$

$$etc. \quad (77)$$

A base element of length n can be associated with a n -simplex with vertices on the events in the element.

Single event simplices $((a))$ and a null simplex $(())$ are included in the algebra.

Multiply by cancelling out any common events with appropriate sign factors. To get the sign right, permute the events until the common ones are at the end of the first set and at the start of the second in the opposite sense. The elements can now be multiplied with the same rule as for the open string. The same parity rules as for closed string apply. I.e. only cancellations of an odd number of events is permitted.

The Lie product of two base elements can only be non-zero if they have an odd number of events in common. e.g.

$$[((1|2|3))((2|3|4))]_{\pm} = 0 \quad (78)$$

$$[((1|2|3|4))((4|3|2|5))]_{\pm} = ((1|5)) \quad (79)$$

This defines real and complex super-lie algebras which will be called *simplex*(0| N , \mathbf{R}) and *simplex*(0| N , \mathbf{C}). These Lie algebras are finite dimensional with dimension 2^N .

An adjoint can be defined on the complex super-algebra in the usual way

$$\Xi = \sum \xi^C C \quad (80)$$

$$\Xi^\dagger = \sum \bar{\xi}^C i^{par(C)} C^T \quad (81)$$

$$= \sum \bar{\xi}^C i^{\text{len}(C)} C \quad (82)$$

If we take the sub-algebra of elements of $\text{simplex}(0|N, \mathbf{C})$ for which

$$\Xi^\dagger = -\Xi \quad (83)$$

then this can be written in terms of their components as

$$\bar{\xi}^C = -i^{\text{len}(C)} \xi^C \quad (84)$$

So

$$\xi^C = \phi^C \exp(i[\pi/4]\text{len}(C)) \quad (85)$$

With ϕ^C being real. If we use these as components writing,

$$\Xi = \sum \phi^C C_R \quad (86)$$

$$C_R = \exp(-i[\pi/4]\text{len}(C)) C \quad (87)$$

It can be checked that the basis on C_R has the same multiplication rules as the basis on C except for an extra minus sign when the number of common events cancelled is 3 mod 4 just as in the algebra $\text{closed}_\pm(0|N, \mathbf{R})$. This is the group $\text{simplex}(0|N)$.

The representations of these groups are families of fully antisymmetric tensors. The Lie algebras are finite dimensional and it is therefore an interesting exercise to determine how they correspond to the classification of semi-simple Lie-algebras by factorising into well known compact groups.

An important remark about the simplex groups is that they have a resemblance to the event-symmetric spinor models which can be seen when their components are written as families of alternating tensors. In fact it is not difficult to see that they are generated by the Clifford algebras for N dimensional space.

A matrix representation of the algebra can be constructed using Gamma matrices which have size $2^{N/2} \times 2^{N/2}$ provided n is even. In this representation a mapping between the basic elements is defined by

$$((a)) \rightarrow \gamma_a \quad (88)$$

The gamma matrices satisfy the anticommutation relations,

$$\gamma_a \gamma_b + \gamma_b \gamma_a = 2\delta_{ab} \quad (89)$$

The full algebra is generated from the linear span of all 2^N possible products of the matrices e.g.

$$((a|b)) \rightarrow \gamma_a \gamma_b \quad (90)$$

The null simplex maps onto the identity matrix.

Since these are all linearly independent matrices with 2^N matrix elements it follows that the algebra over the complex numbers is isomorphic to the full matrix algebra $M(2^{N/2}, \mathbf{C})$. However, we are interested in the Z_2 graded algebra where the parity is given by the size of the simplex. It is possible to construct the gamma matrices so that they all have elements in only the upper right and bottom left quadrants. The grading then maps the algebra onto the super matrix algebra $M(L|L, \mathbf{C})$, where $L = 2^{N/2-1}$. It follows that the Lie-superalgebra formed from the graded anticommutators is just the super-symmetric affine algebra and,

$$\text{simplex}(N, \mathbf{C}) \simeq gl(L|L, \mathbf{C}) \quad (91)$$

while the adjoint defined on the signature algebra corresponds to the usual adjoint on supermatrices so,

$$\text{simplex}(N) \simeq u(L|L) \quad (92)$$

From this it is possible to construct and understand the invariants of the algebra as invariants of the matrix super-groups. These are functions of the supertrace of powers of the matrices.

The first order invariant turns out not to be the component corresponding to the null simplex as you would expect. Instead it corresponds to the simplex formed from all the N events,

$$U = ((1, 2, \dots, N)) \quad (93)$$

This and higher order invariants seem to have anything but a local nature since they are sums over products of simplices which include all events but which have no event in common.

A second order invariant from the trace of the square is a sum over products of two component tensors which have no event in common. This seems to be just the opposite of what we want for a local theory but if we define a relationship between a simplex and a dual simplex as follows

$$\Xi^* = U\Xi \quad (94)$$

Then the model is local.

It is interesting to compare this incomplete study of symmetries on simplices with earlier work of a similar nature. Finkelstein and Rodriguez also noted the importance of Clifford algebras in this context [13, 15, 17]. The ideas presented here were derived independently but the concurrence is important. It is possible that the supersymmetry described here might lead to further developments in this area.

There is an alternative interpretation of the simplex model which is very instructive. The Clifford algebra can be compared directly with the Fock space of a fermi gas (see e.g. [90]). The antisymmetric tensors are then viewed as antisymmetric wavefunctions describing fermion occupancy and the dual mapping can be interpreted as a hole or anti-particle state. A fermi gas is already a second quantised system in quantum field theory and the quantisation procedure applied here is tantamount to a third quantisation.

Duality

The matrix models, when interpreted as event-symmetric, show quite clearly how space-time symmetry and internal gauge symmetry could be unified. This has always been regarded as the final goal which must be scored to unify all physics, but why stop there? There are other symmetries which are often overlooked. Many particle systems are invariant under exchange of identical particles. The wavefunction is symmetric for bosons and antisymmetric for fermions. Since quantum field theory came to prominence this symmetry has been demoted. It seen as a symmetry only of the quantum field and not a true symmetry of the classical Lagrangian like the gauge symmetries. Unification seems to be out of the question.

Is this conclusion justified? I would object. After all there is no classical world, the $\hbar \rightarrow 0$ limit does not exist because changing \hbar only rescales our units of measurement. The universe is a quantum one and invariance under particle exchange is as good a symmetry as any other. Furthermore, the distinction between classical and quantum fields can become blurred. This is dramatically demonstrated by the unity of dualities in string theory [95] which is apparently a duality between the classical and quantum worlds [96]. This classical/quantum duality even manifests itself even in the simple matrix models. Are the two dimensional triangulated manifolds, which arises as the perturbation theory of a matrix model, to be interpreted as the two dimensional classical space-time of a 2-dimensional quantum gravity or the world sheet of a string which are the Feynman diagrams of a $c < 1$

string theory? This kind of duality where the Feynman diagrams of one model become the classical configurations of another are quite common in event-symmetric models.

There is also an analogy between particle systems and event symmetric systems which was exploited in the molecular models. It is now time to ask if this could be more than just an analogy. Could there be a duality between the symmetric group as it acts on space-time events and the symmetric group acting on identical particles? The simplex model shows most clearly that this is viable because it has a dual interpretation as a third quantisation of a fermi gas and an event-symmetric space-time model. Since string models are likely to be quite closely related to this one, this greater unification may be possible.

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